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Inelastic static analysis to evaluate the ductility of coupled walls in a tall building

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ABSTRACT

Inelastic static (push-over) analysis is used to evaluate the ductility of coupled reinforced concrete walls in a 32-story (430 ft high) office tower. The initial stiffness of the structure is controlled by the flexural stiffness of the highly-coupled walls acting as a solid core. After the coupling beams yield, the structure is very flexible as the four elastic walls act separately, and the structure reaches the maximum displacement demand prior to any inelastic action in the walls. The geometry of the walls are such that very small compressive strains would occur if there was yielding of the walls thereby eliminating the need for any confinement of concrete. As the non-linear elastic deformations of the individual walls play an important role in the response of the structure, an accurate tri-linear moment-curvature model, that includes the effect of tension-stiffening, was used.

INTRODUCTION

The majority of high-rise buildings designed and constructed in British Columbia have concrete walls to resist lateral loads due to earthquakes. Compared to moment-resisting frames, concrete walls are more economical to construct and impose less architectural restrictions. Concrete walls provide excellent drift control and are easily provided with adequate ductility by ensuring that the well defined "plastic hinge" regions near the base of the walls are able to undergo the required inelastic deformations.

The Canadian concrete code first introduced a rational approach for evaluating the ductility of walls in the 1984 edition (CSA 1984). Based on the assumption that inelastic deformations of isolated walls result from concentrated rotations near the base, and that rotation can be converted to average curvature using an effective plastic hinge length (Paulay and Uzumeri, 1975), the code defined curvature demand in a ductile (R = 3.5) wall is given in Fig. 1. The curvature ductility demand for a nominally ductile wall is proportionally less due to the reduced inelastic displacement demand. For simplicity, the influence of wall height was not included in the Canadian code provisions.

The Uniform Building Code introduced a rational procedure for evaluating the ductility of walls in the 1997 edition (UBC 1997). The general method requires that confinement be provided in areas where the concrete compressive strains are calculated to exceed 0.0030. While this requirement is conceptually simple, the implementation involves: (i) determining displacements due to specified ground motions accounting for non-linear behaviour of the building, (ii) determining curvature of the wall section at each location of potential flexural yielding, and (iii) using a strain compatibility analysis of the wall cross section to determine the compressive strains resulting from these curvatures. The simplified procedure in UBC for isolated cantilever walls is similar to the Canadian code procedure except that it includes the effect of wall height.

In the absence of any published recommendations on how to evaluate the ductility of coupled-walls, the 1984 and subsequent edition of the Canadian code assumes the curvature demand in fully coupled-walls is as given in Fig. 1 with the length l_w taken as the overall length of the coupled system. The validity of this approach is yet to be confirmed. The SEAOC (1996) Blue Book commentary on the UBC provisions states that a simplified approach cannot be used for coupled-walls because the moment gradient and associated curvature-displacement relationship will be much different from those for a cantilever wall. It also outlines the general approach in more detail as follows: (i) select plastic hinge locations; (ii) develop nonlinear moment-curvature relationships for each critical section; (iii) undertake plastic or inelastic analysis such as a static pushover

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analysis to verify plastic hinge locations, account for redistribution of forces and deformations of wall system, determine hinge rotations, and ensure brittle shear failure does not occur; (iv) convert plastic hinge rotation to curvature; (v) convert curvature to strains. The SEAOC Blue Book indicates that the intent of the general method in UBC is to allow and encourage the use of a range of detailed analyses to evaluate ductility, and "*from the experience and results that will be gained from these analyses, it will be possible to improve upon the simplified procedures for future requirements.*" This paper presents the results from an analysis of highly-coupled walls in a tall building. The analysis results provide valuable insight into the ductile behaviour of such structures.

NONLINEAR MOMENT-CURVATURE RELATIONSHIPS

A typical nonlinear moment-curvature relationship for a wall section is shown in Fig. 2. Walls are typically lightly reinforced and have low axial compression (up to about $0.20 f_c' A_g$). The concrete stresses are in the predominantly linear range and as a result, the moment-curvature relationship is essentially tri-linear. Little error results from assuming a perfectly tri-linear relationship as shown in Fig. 2.

As the applied bending moment is increased beyond the initial linear region, the flexural stiffness reduces. Note that the relationship shown in Fig. 2 is for a previously cracked wall. That is, the concrete tension stress-strain relationship used to predict the relationship is such that the maximum tension stress in concrete is much less than the cracking strength. Due to bond, the concrete between cracks does still contribute significantly to stiffening the reinforcement at strains much larger than the cracking strain, an effect known as tension-stiffening. The average tensile stresses in concrete do not contribute to the maximum capacity which is controlled by yielding of reinforcement at a crack.

Fig. 3 shows two series of moment-curvature relationships. Fig. 3(a) shows the relationships for a series of wall sections with constant concrete geometry and axial load, but varying amount of reinforcement. Fig. 3(b) is for a series with constant concrete geometry and reinforcement, but varying axial load. The initial slope of all relationships depends primarily on the concrete geometry, and is approximately equal to EI_g. The secondary slope depends primarily on the amount of reinforcement, and is approximately parallel to the well known stiffness of a cracked linear section EI_{cr}. The bending moment at which the transition from initial to secondary slope occurs depends primarily on the level of axial compression in the wall section. This bending moment is referred to as M_L in Fig. 2.

Fig. 3(c) shows a typical relationship between the applied axial load P and the ultimate flexural capacity of the wall section M_u, as well as the bending moment that defines the end of the initial linear range M_L. The $P–M_l$ interaction is determined by fitting a family of tri-linear curves to a family of actual moment-curvature relationships. A requirement of equal area under the moment-curvature relationships is used to fit the curves. Similar to $P–M_U$ interaction diagrams below the balanced failure point where failure is controlled by reinforcement yielding, the $P–M_L$ interaction diagram is approximately linear.

PUSH-OVER PROGRAM

A linear two-dimensional plane frame program was modified to account for non-linear stiffness of members. The tri-linear model described above is used for the moment-curvature relationships. For each wall section, that has constant concrete geometry and reinforcement amount, the family of idealized moment-curvature relationships is input in terms of two slopes EI_g and EI_{cr} and two non-linear interaction diagrams $P–M_L$ and $P–M_U$.

At each load step, an iterative process is used to determine the correct effective member stiffness. The effective stiffness is determined as follows: the bending moment distribution along the member length is determined from the member end forces (no member forces are applied); the curvature distribution along the length of the member is determined from the tri-linear moment-curvature relationship; the equivalent member stiffness EI_e is that which gives the equivalent total rotation along the member length.

When the bending moment at one end of a member reaches the yield (ultimate) capacity, a hinge is inserted in the structure and a joint moment is applied to simulate the plastic moment resisted by the member. During a typical push-over analysis, the vertical (gravity) loads on the structure are held constant, while the lateral loads are increased in small increments. A typical load step is 5%. When the target displacement has been reached, or the structure becomes unstable, the analysis is stopped.

EXAMPLE

To study the behaviour of highly-coupled walls, the reinforced concrete core of a 32-story structural steel office tower was analysed using the inelastic static (push-over) method. The overall height of the structure above grade is 430 ft. Fig. 4(a) shows a three-dimensional view of the coupled-wall core, and details of the core cross-section are given in Fig. 5. Due to the high axial compression in the core walls from the tributary floor area, the flexural capacity of the walls is adequate with only the minimum vertical reinforcement in all walls.

The structure was analysed in the long direction. While openings were also added in the short direction of the core, these are not shown in the figure below as they are not relevant to the present analysis. Fig. 4(b) shows the two-dimensional model used to analyse the structure in one direction. The walls were modelled as column elements located at the gross section centroids. The coupling beam elements were connected to the "columns" using infinite stiffness extensions.

Prior to undertaking the two-dimensional non-linear static analysis, a three-dimensional linear dynamic analysis of the structure was done. Fig. 4(c) shows the coupling beam shears determined from the modal analysis. Due to the numerous coupling beams over the height of the structure, the degree of coupling was found to be about 94%. Also shown in Fig. 4(c) are the assumed plastic capacities of the diagonally reinforced coupling beams.

The linear dynamic analysis was also used to establish the maximum inelastic response displacement for the structure during an earthquake that has a 10% probability of occurrence in 50 yrs and during an earthquake that has a 2% probability of occurrence in 50 yrs (times a factor of 2/3). These maximum displacement demands of the structure were 34 in. and 50 in., respectively.

Fig. 6 shows the results from the push-over analysis that was done for the first-mode distribution of forces (see Fig. 4b). The first coupling beam yielded at a base shear of about 1200 kips. When the base shear was about 2500 kips, many of the coupling beams had yielded and the structure became noticeably softer. After all the coupling beams yielded, the remaining structure consisted of four elastic walls that were very flexible. The walls did not yield at the base until the structure had deflected to about 65 in., which is a considerably larger displacement than is expected to occur during the 2500 yr earthquake. Prior to the walls yielding, the effective stiffness of the walls in the lower portion of the structure had reduced to $0.3EI_g$ as shown in the Table inset in Fig. 6.

The significance of the behaviour described above is that during any foreseeable earthquake, the highly-coupled core walls will dissipate energy by yielding of the diagonally reinforced coupling beams (a very stable mechanism), and the walls will remain elastic. It is very desirable to have a system in which the vertical gravity load resisting elements remain elastic. Even though the walls are not expected to become inelastic, the geometry of the walls are such that the moment-curvature analysis indicates that very small compressive strains will result from large inelastic curvatures of the walls.

It is interesting to note the importance of accurately modelling the flexural deformations of the elastic walls. It is usually conservative to assume a lower than actual stiffness, as the displacement demand is correspondingly increased. However, assuming lower than actual flexural stiffnesses of the individual walls in this case does not significantly influence the initial stiffness of the highly coupled wall system, but results in significantly larger elastic deflections prior to yielding of the walls.

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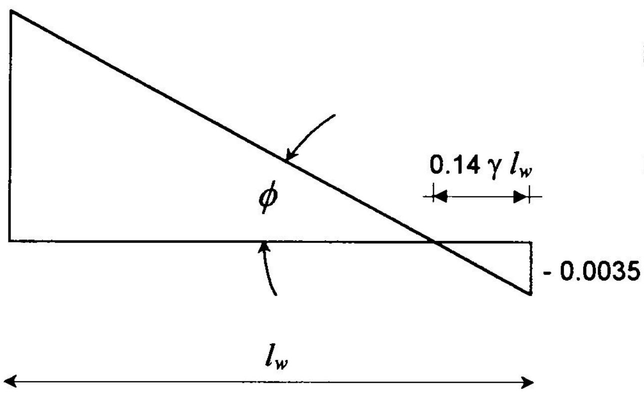


Fig. 1–CSA code curvature demand for ductile walls.

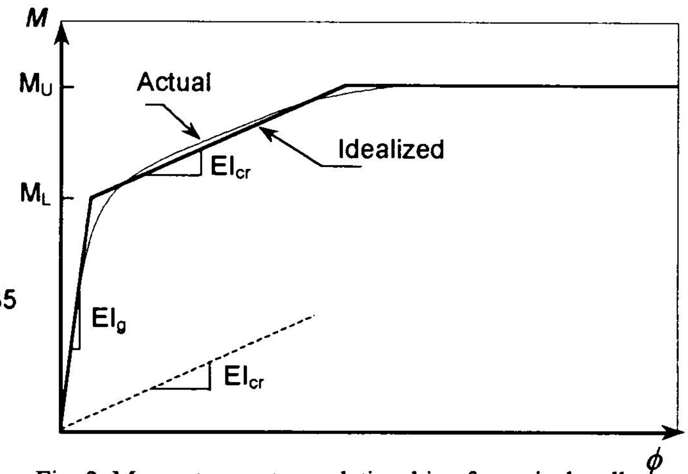


Fig. 2–Moment-curvature relationship of a typical wall.

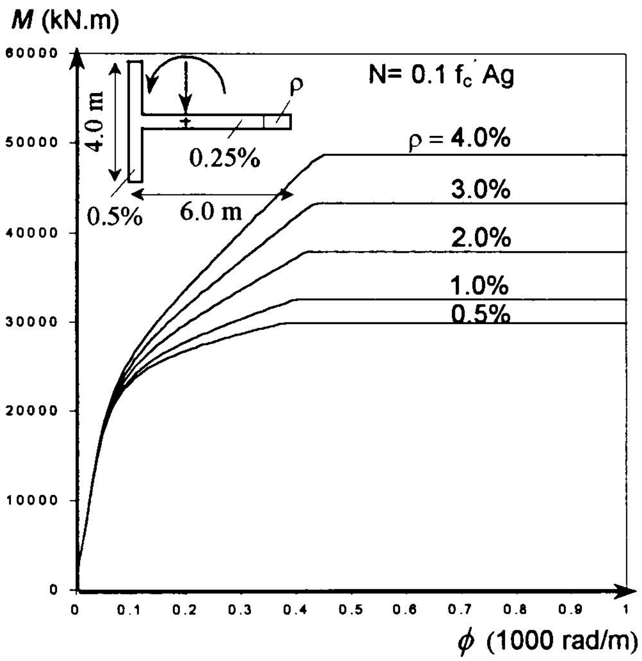


Fig. 3(a)–Influence of reinforcement amount.

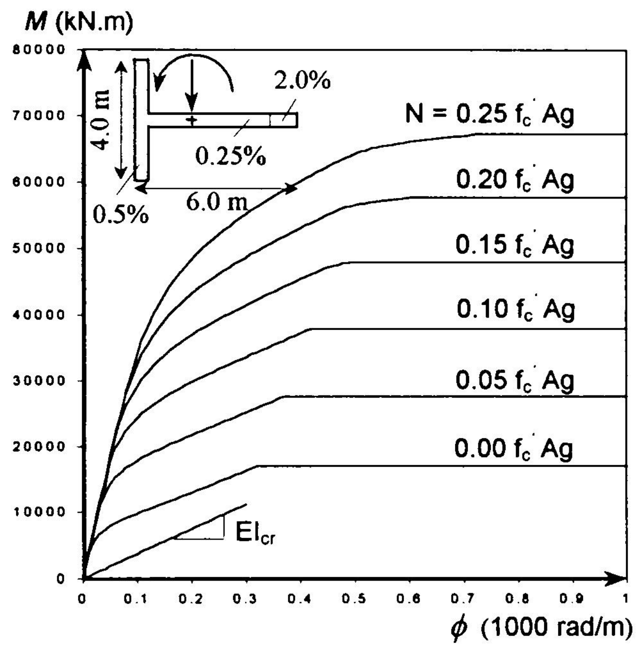


Fig. 3(b)–Influence of axial compression.

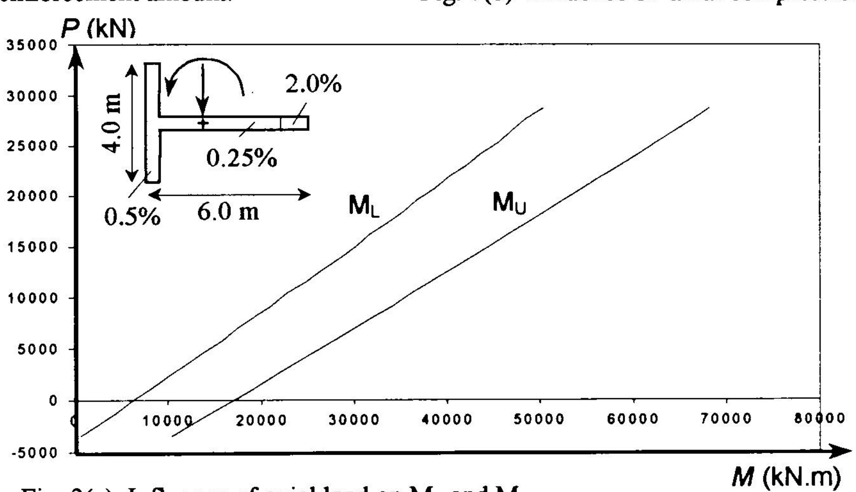
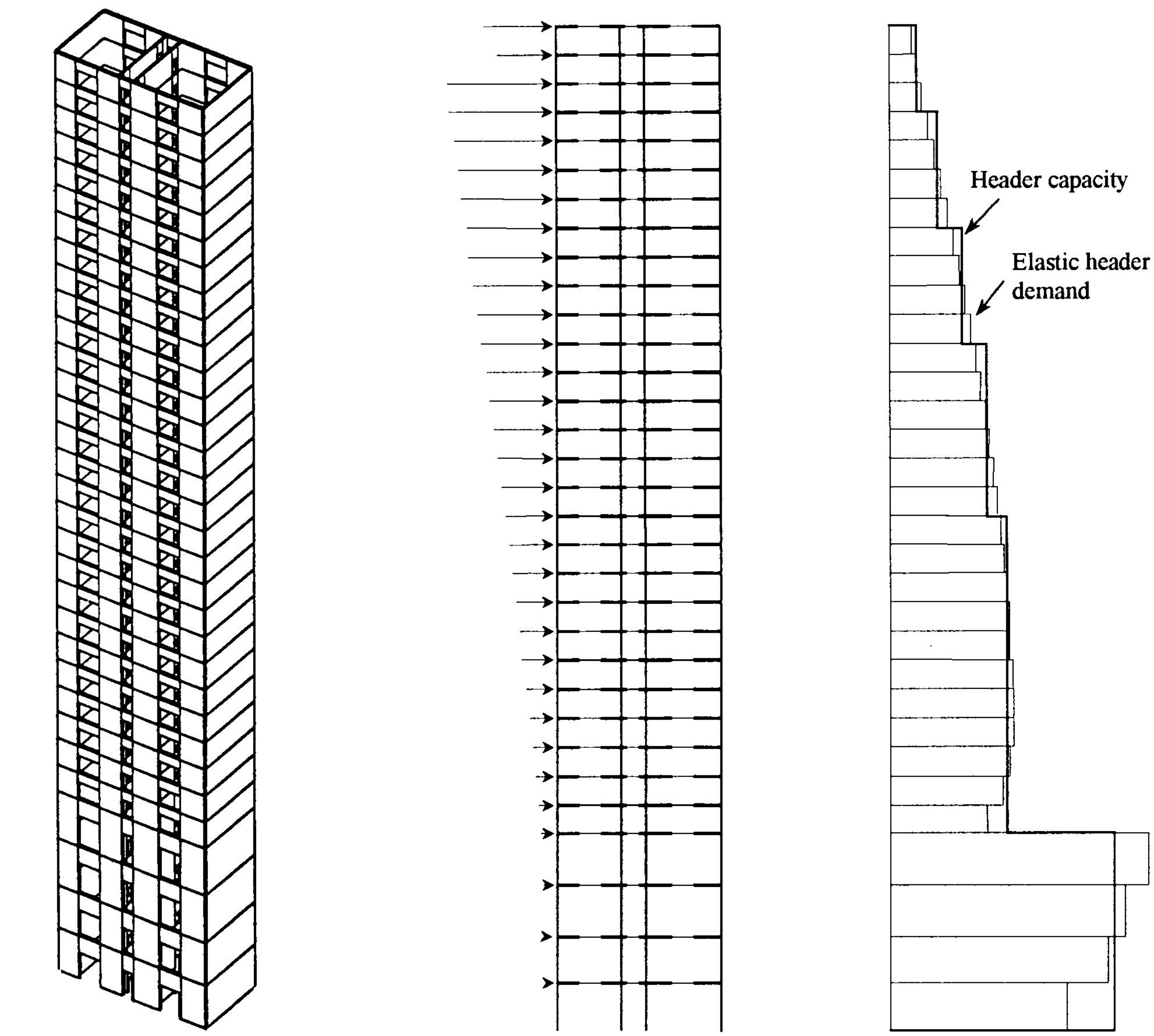


Fig. 3(c)–Influence of axial load on M_L and M_U.



(a) Isometric view of core. (b) Plane frame model. (c) Coupling beam shears.

Fig. 4–Example problem: highly-coupled walls in a 32 story office tower.

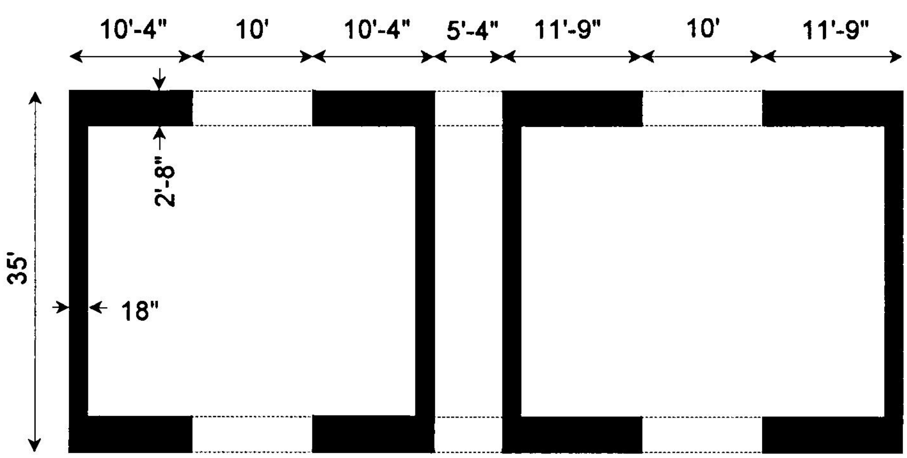
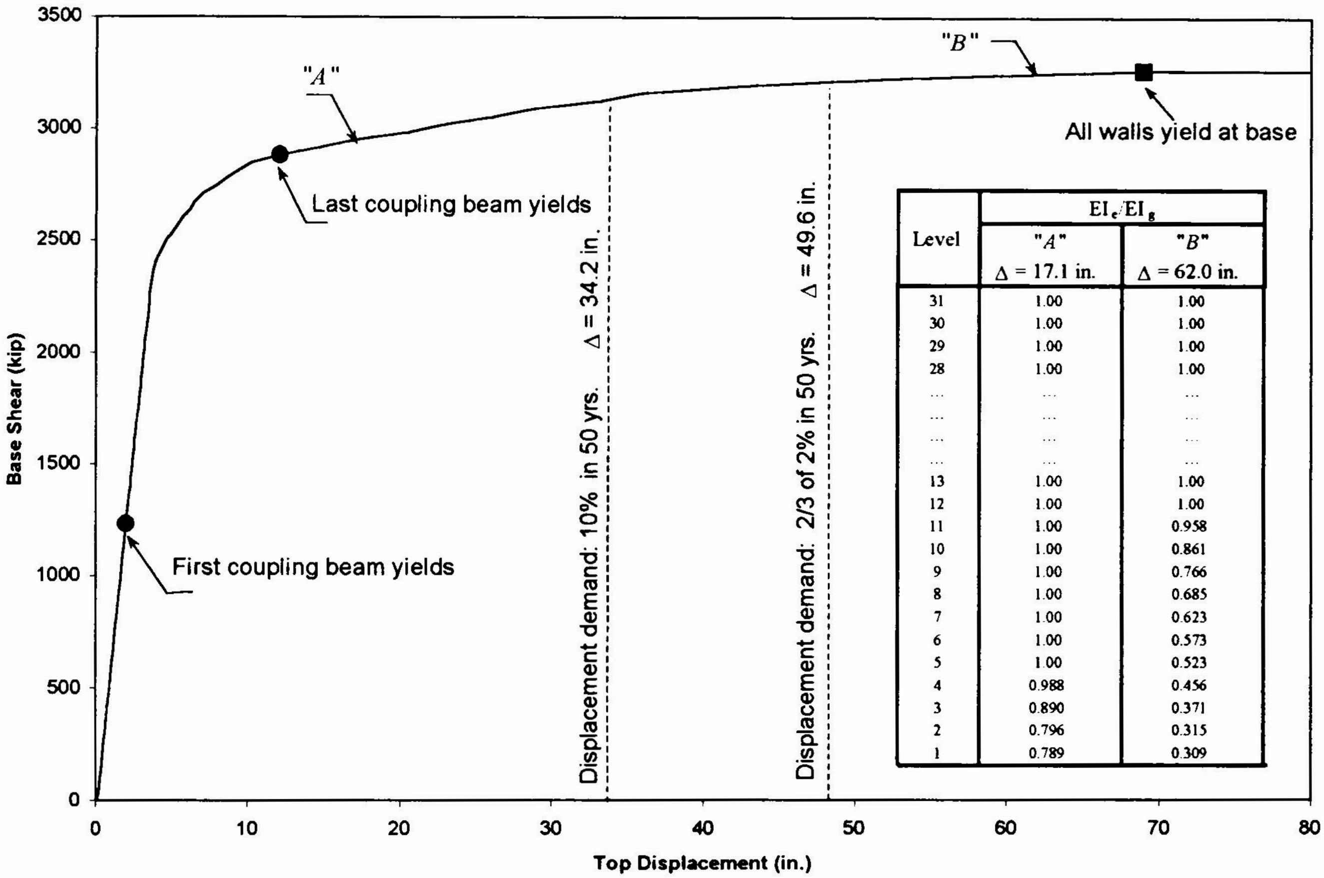


Fig. 5–Cross section of the coupled walls.



Level	EI_e/EI_g	
	"A" Δ = 17.1 in.	"B" Δ = 62.0 in.
31	1.00	1.00
30	1.00	1.00
29	1.00	1.00
28	1.00	1.00
...	...	...
...	...	...
...	...	...
...	...	...
13	1.00	1.00
12	1.00	1.00
11	1.00	0.958
10	1.00	0.861
9	1.00	0.766
8	1.00	0.685
7	1.00	0.623
6	1.00	0.573
5	1.00	0.523
4	0.988	0.456
3	0.890	0.371
2	0.796	0.315
1	0.789	0.309

Fig. 6–Predicted load-deformation response of highly-coupled walls.